V. *A Propoſition of General Uſe in the Art of Gunnery, ſhewing the Rule of Laying a Mortar to paſs, in order to ſtrike any Object above or below the Horizon. By* E. Halley.

IT was formerly the Opinion of thoſe concerned in Artillery, that there was a certain requiſite of Powder for each Gun, and that in Mortars, where the Diſtance was to be varied, it muſt be done by giving a greater or leſſer Elevation to the Piece. But now our later Experience has taught us that the ſame thing may be more certainly and readily performed by increaſing and diminiſhing the quantity of Powder, whether regard be had to the Execution to be done, or to the Charge of doing it. For when Bombs are diſcharged with great Elevations of the Mortar, they fall too Perpendicular,and bury themſelves too deep in the Ground, to do all that damage they might, if they came more Oblique, and broke upon or near the Surface of the Earth; which is a thing acknowledged by the beſieged in all Towns, who unpave their Streets,to let the Bombs bury themſelves, and thereby ſtifle the force of their Splinters. A Second Convenience is, that at the extream Elevation, the Gunner is not obliged to be ſo Curious in the Direction of his Piece, but it will ſuffice to be within a Degree or two of the truth; whereas in the other Method of Shooting he ought to be very curious. But a Third, and no leſs conſiderable Advantage is, in the ſaving the King's Powder, which in ſo great and ſo numerous Diſcharges, as we have lately ſeen, muſt needs amount to a conſiderable value. And for Sea-Mortars, it is ſcarce Practicable otherwiſe to uſe them, where the agitation of the Sea continually changes the direction of the Mortar, and would render the Shot very

very uncertain, were it not that they are placed about 45 Degrees Elevation, where ſeveral Degrees above or under makes very little difference in the Effect.

In *Numb.* 179. of theſe Tranſactions, I conſidered and demonſtrated all the Propoſitions relating to the motion of Projectiles, and gave a Solution to this Problem, *viz.* *To hit an Object above or below the Horizontal Line with the greateſt certainty and leaſt force*, as may be ſeen in that Tranſaction, *p.* 16. & 17. That is, that the Horizontal diſtance of the Object being put $= b$, and the Perpendicular height $= h$, the Charge requiſite to ſtrike the Object with the greateſt Advantage, was that which with an Elevation of 45° would caſt the Shot on the Horizontal Line to the diſtance of $\sqrt{bb + hh} + h$ when the Object was above the Horizon; or if it were below it, the Charge muſt be leſſer, ſo as to reach on the Horizon, at 45° Elevation, no greater a diſtance than $\sqrt{bb + hh} - h$, that is, in the one caſe, the Sum of the Hypothenuſal diſtance of the Object from the Gun and the Perpendicular height thereof above the Gun; and in the other caſe, when the Object is below the Horizon, the difference of the ſame, *per* 47. 1. *Eucl.* And I then ſhewed how to find the Elevation proper for the Gun ſo charged, *viz.* As the Horizontal diſtance of the Object, to the Sum or difference of the Hypothenuſal diſtance and Perpendicular height : : So Radius to the Tangent of the Elevation ſought. But I was not at that time aware that the aforeſaid Elevation did conſtantly biſect the Angle between the Perpendicular and the Object, as is demonſtrated from the Difference and Sum of the *Tangent* and *Secant* of any Arch being always equal to the *Tangent* and *Cotangent* of the half Complement thereof to a Quadrant. Having diſcovered this, I think nothing can be more compendious, or bids fairer to compleat the Art of Gunnery, it being as eaſie to ſhoot with a Mortar at any Object on demand, as if it were

on the Level; neither is there need of any Computation, but only ſimply laying the Gun to paſs, in the middle Line between the Zenith and the Object, and giving it its due Charge. Nor is there any great need of Inſtruments for this purpoſe: For if the Muzle of the Mortar be turned truly Square to the Bore of the Piece, as it uſually is or ought to be, a piece of Looking-glaſs Plate applyed parallel to the Muzle, will by its Reflection give the true Poſition of the Piece; the Bombardeer having no more to do, but to look Perpendicularly down on the Looking-glaſs, alongſt a ſmall Thread with a Plumbet, and to raiſe or depreſs the Elevation of the Piece, till the Object appear reflected on the ſame Point of the *Speculum*, on which the Plumbet falls; for the Angle of Incidence and Reflection being equal, in this caſe a Line at Right Angles to the *Speculum*, as is the *Axis* of the Chaſe of the Piece, will biſect the Angle between the Perpendicular and the Object, according as our Propoſition requires. So that it only remains by good and valid Experiments to be aſſured of the force of Gunpowder, how to make and conſerve it equal, and to know the effect thereof in each Piece; that is, how far differing Charges will caſt the ſame Shot out of it; which may moſt conveniently be engraven on the outſide thereof, as a ſtanding Direction to all Gunners, who ſhall from thence forward have occaſion to uſe that Piece: And were this matter well aſcertained, it might be worth the while to make all Mortars of the like Diameter, as near as may be alike in length of Chaſe, Weight, Chamber, and all other Circumſtances.

This Diſcovery that the utmoſt Range on an inclined Plane is, when the *Axis* of the Piece makes equal Angles with the Perpendicular and the Object, compared with what I have demonſtrated of the ſame Problem in the aforeſaid *Numb.* 179. does lead to and diſcover two very

ready Theorems; the one to find the greatest Horizontal Range at 45° Elevation, by any Shot made upon any inclined Plane, with any Elevation of the Piece whatsoever: And the other to find the Elevations proper to strike a given Object, with any force greater than what suffices to reach it with the aforesaid middle Elevation. Both which being performed by one single Proportion, may be very serviceable to such as are concerned in the Practice of Gunnery, but are unwilling to trouble themselves with tedious and difficult Rules. The two Propositions are these.

PROP. I.

A Shot being made on an Inclined Plane, having the Horizontal Distance of the Object it strikes, with the Elevation of the Piece, and the Angle at the Gun between the Object and the Perpendicular: to find the greatest Horizontal Range of that Piece, laden with the same Charge; that is, half the *Latus rectum* of all the *Parabolæ* made with the same *impetus*.

RULE.

Take half the Distance of the Object from the *Nadir*, and take the difference of the given Elevation from that half; the Versed Sine of that difference substract from the Versed Sine of the Distance of the Object from the *Zenith*: Then shall the difference of those Versed Sines be to the Sine of the Distance of the Object from the *Zenith*, as the Horizontal Distance of the Object struck to the greatest Horizontal Range at 45°.

PROP. II.

Having the greatest Horizontal Range of a Gun, the Horizontal Distance and Angle of Inclination of an Object to the Perpendicular, to find the two Elevations necessary to strike that Object.

RULE.

Halve the Distance of the Object from the *Nadir*, this half is always equal to the half Sum of the two Elevations we seek. Then say, *As the greatest Horizontal Range is to the Horizontal Distance of the Object: So is the Sine of the Angle of Inclination or Distance of the Object from the Perpendicular, to a fourth Proportional; which fourth being substracted from the Versed Sine of the Distance of the Object from the* Zenith, *leaves the Versed Sine of half the difference of the Elevations sought; which Elevations are therefore had by adding and substracting that half difference to and from the aforesaid half Sum.*

I shall not need to speak of the facility of these Solutions, I shall only observe that they are both General, without Exception or Caution, and derived from the knowledge that these two Elevations are equidistant above and below the Line bisecting the Angle between the Object and the *Zenith*.
